GROWTH BY THE CURVE, NOT THE CUT
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Abstract—A norm-referenced growth model using growth percentiles is proposed. A caveat of criterion-referenced growth models are they tend to only capture the growth of students whose achievement are around the cut points. In the proposed model, student growth is explored as current achievement relative to student with identical prior achievement.

Objectives—The basic theory of including growth models in school accountability systems is that effective schools produce more student growth (an increase in student achievement per year) than ineffective schools. This exceptional increase in student achievement is causally attributed to the school or teachers instructing the students and adding value to their students.

The current Iowa growth model used for Adequate Yearly Progress (AYP) under the No Child Left Behind (NCLB) legislation uses criterion-referenced cut points to measure student growth in reading and math from year to year. Based on these cut points, students are placed into one of seven possible achievement levels. The table below shows the achievement levels with the corresponding national percentile rank range for the Iowa Test of Basic Skills (ITBS), grades 3-8, and the proficiency status of achievement levels.

Table 1—Current Iowa Growth Model Achievement Levels*

<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Weak</th>
<th>Low Marginal</th>
<th>High Marginal</th>
<th>Moderate</th>
<th>Skilled</th>
<th>Accomplished</th>
<th>Distinguished</th>
</tr>
</thead>
<tbody>
<tr>
<td>National Percentile Rank Range</td>
<td>1% - 9%</td>
<td>10% - 32% (depending on grade level and subject)</td>
<td>29% - 40% (depending on grade level and subject)</td>
<td>41% - 75%</td>
<td>76% - 89%</td>
<td>90% - 94%</td>
<td>95% - 99%</td>
</tr>
<tr>
<td>Proficient/Non-proficient</td>
<td>non-proficient</td>
<td>non-proficient</td>
<td>non-proficient</td>
<td>proficient</td>
<td>proficient</td>
<td>proficient</td>
<td>proficient</td>
</tr>
</tbody>
</table>

Source: Iowa Department of Education, Iowa NCLB Growth Model.
*Columns are scaled (not accurately) to represent range of National Percentile Rank Scores contained in each achievement level.

If a student moves up from one achievement level to a higher achievement level the next year, that student shows growth.¹ A caveat of using a cut-point based growth model is it tends to only capture the growth of students whose achievement points are around the

¹Only students who are in a non-proficient achievement level and move up to a higher non-proficient achievement level count for meeting growth in Iowa’s AYP Federal accountability plan.
cut points. The moderate achievement level contains the widest range of national percentile ranks. Using this model, if a student scores in the 41st percentile in grade three, that student will have made astronomical gains—35 percentile ranks to achieve growth in his/her fourth grade year based on our current level cut points. However, a student who scores in the 75th percentile in that same third grade cohort could merely maintain average learning and with the help of standard error, score in the 76th percentile his/her fourth grade year and achieve growth.

An alternative approach would be to assess growth on norm-referenced criteria. Based on the student’s achievement last year, where would we expect (based on the student’s peer group) the student’s achievement to be this year? Students who achieve above that average show growth. (Showing more than one year of achievement in a year’s time compared to their peers.) This paper will explore a methodology for a norm-referenced growth model in order to reward Iowa schools for all students who are showing growth, not just students crossing set cut points.

**Design and Methods**—Student growth should be examined as a student’s “current achievement relative to their academic peers—those student with identical prior achievement” (Betebenner, 2008, p. 4). The distribution of current student growth can be broken down into percentiles with the 50th percentile representing average current achievement, conditional on past achievement. Students with a growth percent less than 50 have not done well, while students above 50 have done well compared to their academic peers. The higher a student’s growth percentile, the more growth they have made compared to their peers. For example, students with a growth percentile of 20 have grown more than only 20 percent of their peers, while students with a growth percentile of 80 have grown more than 80 percent of their peers.

\[
\text{Student Growth Percentile} = \Pr(\text{Current Achievement} \mid \text{Past Achievement}) \times 100
\]

Student growth percentile distributions (or densities) are estimated using quantile regression (Koenker and Hallock, 2001; Koenker, 2005). Quantile regression models estimate multiple distributions of the response variable Y, dependent on position of the outcome. Conditioning on a covariate x, the \( \tau \)th conditional quantile function, \( Q_y(\tau \mid x) \), is given by

\[
Q_y(\tau \mid x) = \arg\min_{\beta \in \mathbb{R}^p} \sum_{i=1}^{n} \rho_\tau(y_i - x_i' \beta)
\]

If \( \tau = 0.5 \), then the estimate conditional quantile line is the median regression line.

Unique state student identifiers (known as Iowa student numbers) are used to match ITBS test scores of students who took the ITBS in both the 2007-2008 and 2008-2009 school years. Third grade test scores are used to predict fourth grade test scores in reading, math, and science. Fifth grade test scores are used to predict sixth grade scores, and seventh grade test scores are used to predict eighth grade scores in reading, math and science. \(^2\)

\(^2\)Predictions could be made for all progressions of grades. However, to keep this paper brief, only three grade progressions were included.
Results—The figures below (1-3) show fourth grade growth distributions based on third grade achievement for reading, math, and science. These distributions can be used to find the growth percentiles (also known as quantiles) for an individual student by first, finding his or her achievement score for the predictor year/test on the horizontal axis and draw a line up. Then find his or her achievement score for the second year/test on the vertical axis and draw a line to the right. The approximate quantile line where the two lines converge is the student’s growth percentile. A student that performs at the \( n^{th} \) growth percentile (quantile) out-performed \( n \) percent of his or her academic peers (students who got the same test score last year).

Students who perform poorly the first year of the test are more likely to grow than students who perform well. Students who start at the upper end of the test either do not have much room to grow or the test cannot measure their growth. The growth distributions for reading, math, and science at fourth grade appear similar. However, the growth percentiles (quantiles) for science are more spread out than for reading and math. The median quantile for science also has a flatter slope.

Figure 1—Fourth Grade Reading Growth Distribution

![Fourth Grade Reading Growth Distribution](image_url)
Figure 2—Fourth Grade Math Growth Distribution

![Fourth Grade Math Growth Distribution](image)

Source: Iowa Department of Education, AYP Files.

Figure 3—Fourth Grade Science Growth Distribution

![Fourth Grade Science Growth Distribution](image)

Source: Iowa Department of Education, AYP Files.
Growth distribution tables for sixth and eighth grade in reading, math, and science are located in Appendix A.

**Significance**—Once student growth percentiles are calculated for all grades, a lookup table could be created to display student growth percentiles through Iowa’s data warehouse. This information would be invaluable to Iowa teachers, administrators, and parents. Growth percentile could also be aggregated up to the building and district level to inform district staff on performance and to compare buildings/districts with others across the state.

Student growth percentiles not only tell if a student grew, but how much a student grew. A student that performed in the 71\textsuperscript{st} percentile out-performed 71 percent of his/her academic peers (students who got the same test score last year). This type of growth model is much more balanced than a growth model that only recognizing students who move across set cut points as showing growth. It’s time for Iowa to recognize the hard work and progress of students at all achievement points.

**REFERENCES**


APPENDIX

Figure 1-A—Sixth Grade Reading Growth Distribution

Source: Iowa Department of Education, AYP Files.

Figure 2-A—Sixth Grade Math Growth Distribution

Source: Iowa Department of Education, AYP Files.
Figure 3-A—Sixth Grade Science Growth Distribution

![Sixth Grade Science Growth Distribution](image1)

Source: Iowa Department of Education, AYP Files.

Figure 4-A—Eighth Grade Reading Growth Distribution

![Eighth Grade Reading Growth Distribution](image2)

Source: Iowa Department of Education, AYP Files.
Figure 5-A—Eighth Grade Math Growth Distribution

Source: Iowa Department of Education, AYP Files.

Figure 6-A—Eighth Grade Science Growth Distribution

Source: Iowa Department of Education, AYP Files.