Progressions for the Common Core State Standards in Mathematics (draft)

26 December 2011

Grade 7

Chance processes and probability models In Grade 7, students build their understanding of probability on a relative frequency view of the subject, examining the proportion of "successes" in a chance process—one involving repeated observations of random outcomes of a given event, such as a series of coin tosses. "What is my chance of getting the correct answer to the next multiple choice question?" is not a probability question in the relative frequency sense. "What is my chance of getting the correct answer to the next multiple choice question *if I make a random guess among the four choices?*" is a probability question because the student could set up an experiment of multiple trials to approximate the relative frequency of the outcome.• And two students doing the same experiment will get nearly the same approximation. These important points are often overlooked in discussions of probability.[7.SP.5]

• Note the connection with MP6. Including the stipulation "if I make a random guess among the four choices" makes the question precise enough to be answered with the methods discussed for this grade.

7.SP.5 Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Students begin by relating probability to the long-run (more than five or ten trials) relative frequency of a chance event, using coins, number cubes, cards, spinners, bead bags, and so on. Hands-on activities with students collecting the data on probability experiments are critically important, but once the connection between observed relative frequency and theoretical probability is clear, they can move to simulating probability experiments via technology (graphing calculators or computers).

It must be understood that the connection between relative frequency and probability goes two ways. If you know the structure of the generating mechanism (e.g., a bag with known numbers of red and white chips), you can anticipate the relative frequencies of a series of random selections (with replacement) from the bag. If you do not know the structure (e.g., the bag has unknown numbers of red and white chips), you can approximate it by making a series of random selections and recording the relative frequencies.[7.SP.6] This simple idea, obvious to the experienced, is essential and not obvious at all to the novice.• The first type of situation, in which the structure is known, leads to "probability"; the second, in which the structure is unknown, leads to "statistics."

7.SP.6 Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

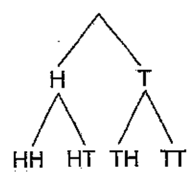
• Examples of student strategies for generalizing from the relative frequency in the simplest case (one sample) to the relative frequency in the whole population are given in the Ratio and Proportional Relationship Progression, p. 11.

A *probability model* provides a probability for each possible non-overlapping outcome for a chance process so that the total probability over all such outcomes is unity. The collection of all possible individual outcomes is known as the *sample space* for the model. For example, the sample space for the toss of two coins (fair or not) is often written as {TT, HT, TH, HH}. The probabilities of the model can be either *theoretical* (based on the structure of the process and its outcomes) or *empirical* (based on observed data generated by the process). In the toss of two balanced coins, the four outcomes of the sample space are given equal theoretical probabilities of $\frac{1}{4}$ because of the symmetry of the process—because the coins are balanced, an outcome of heads is just as likely as an outcome of tails. Randomly selecting a name from a list of ten students also leads to equally

Different representations of a sample space

HH
HT
TH
TT

	H	T
H	HH	HT
T	TH	TT



All the possible outcomes of the toss of two coins can be represented as an organized list, table, or tree diagram. The sample space becomes a probability model when a probability for each simple event is specified.

likely outcomes with probability 0.10 that a given student's name will be selected.[7.SP.7a] If there are exactly four seventh graders on the list, the chance of selecting a seventh grader's name is 0.40. On the other hand, the probability of a tossed thumbtack landing point up is not necessarily $\frac{1}{2}$ just because there are two possible outcomes; these outcomes may not be equally likely and an empirical answer must be found be tossing the tack and collecting data.[7.SP.7b]

The product rule for counting outcomes for chance events should be used in finite situations like tossing two or three coins or rolling two number cubes. There is no need to go to more formal rules for permutations and combinations at this level. Students should gain experience in the use of diagrams, especially trees and tables, as the basis for organized counting of possible outcomes from chance processes.[7.SP.8] For example, the 36 equally likely (theoretical probability) outcomes from the toss of a pair of number cubes are most easily listed on a two-way table. An archived table of census data can be used to approximate the (empirical) probability that a randomly selected Florida resident will be Hispanic.

After the basics of probability are understood, students should experience setting up a model and using simulation (by hand or with technology) to collect data and estimate probabilities for a real situation that is sufficiently complex that the theoretical probabilities are not obvious. For example, suppose, over many years of records, a river generates a spring flood about 40% of the time. Based on these records, what is the chance that it will flood for at least three years in a row sometime during the next five years?[7.SP.8c]

[7.SP.7a] Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

[7.SP.7b] Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

b Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

[7.SP.8] Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

[7.SP.8c] Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

c Design and use a simulation to generate frequencies for compound events.