

Innovation Configuration Map for Iowa Core Mathematics: The Standards for Mathematical Practice and Teaching Fractions Role: Grade 3-6 Mathematics Teacher

This document includes an Innovation Configuration Map (IC Map) for Iowa Core Mathematics: The Standards for Mathematical Practice (SMP).

The purpose of the IC Map is to:

- Provide a vision of high quality mathematics teaching in regard to each SMP.
- Describe a continuum of specific teacher actions ranging from “unsatisfactory” to “exemplary” for each SMP.
- Provide a structure for individual reflection of one’s current teaching practice regarding each SMP.
- Provide a structure to help individual teachers improve their teaching practice by taking action to move along the continuum to “Exemplary”.
- Provide a structure to measure growth in implementing change in classroom practice.

Directions:

In addition to the IC Map for the Standards of Mathematical Practice, this document also includes empty cells below each set of teacher actions. Use these blank cells to describe specific teacher actions for each SMP in regard to teaching fractions. Describe both “Exemplary” and “Unsatisfactory” actions to contrast best practice and common actions in a traditional classroom.

| Component 1: The teacher structures the learning so students <i>make sense of problems and persevere in solving them.</i> | | | |
|--|---|--|---|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> Challenges students with rich mathematical problems on a daily basis. Poses problems without directions on how to approach the problem. Requires students to make decisions and plan a solution pathway. Builds a community of learners where students hear, share, and judge the reasonableness of strategies and solutions. | <ul style="list-style-type: none"> Engages students in rich mathematical problems on a weekly basis. Presents problems and intervenes when students struggle by asking leading questions. Encourages students to work with a partner or in small groups and helps students determine the reasonableness of strategies and solutions. | <ul style="list-style-type: none"> Involves students in solving routine problems in which they already know a procedure for solving on a daily basis. Explains problems and asks leading questions to help students get started. Directs students to work individually on tasks and encourages students to seek feedback from the teacher on the reasonableness of strategies and solutions. | <ul style="list-style-type: none"> Assigns students computational practice problems on a daily basis. Explains problems and shows students one way to solve problems. Directs students to work individually when solving problems and turn in answers for the teacher to evaluate. |
| <ul style="list-style-type: none"> Begins fraction instruction by posing contextual problems, beginning with Equal Sharing problems with answers greater than one. As a result students create representations to make sense of fractions. Provides students multiple opportunities to solve contextual problems with increasingly difficult number combinations before posing non-contextual open number sentences. Does not provide directions for solving fraction problems, but allows students to solve problems in ways that make sense to them. Facilitates classroom discussions where students share and compare solutions. | | <ul style="list-style-type: none"> Begins fraction instruction with non-contextual problems. Shows and explains premade manipulatives and diagrams to help students identify fractions less than one. Progresses to non-contextual computational problems beginning with addition of fractions with like denominators. Shows students one way to solve fraction problems. Collects and evaluates student answers without classroom discussion. | |

| Component 2: The teacher structures the learning so students <i>reason abstractly and quantitatively</i> . | | | |
|---|--|---|---|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> • Expects students to make sense of quantities and their relationships, asking questions in the process. This involves having students represent contextual situations symbolically and interpret answers in the context of the problem. • Encourages the flexible use of multiple solution strategies when solving problems. | <ul style="list-style-type: none"> • Asks leading questions to help students make sense of quantities and relationships. Asks questions leading to symbolic representations and asks questions to clarify answers. • Encourages a few different solution strategies when solving problems. | <ul style="list-style-type: none"> • Shows students how to represent contextual situations symbolically and interprets answers for students. • Encourages one solution strategy when multiple approaches lead to correct solutions, but accepts multiple strategies. | <ul style="list-style-type: none"> • Assigns students problems to solve with a known procedure. • Accepts one solution strategy when multiple approaches lead to correct solutions. |
| <ul style="list-style-type: none"> • Expects students to make sense of fractions as they represent and solve challenging contextual and non-contextual problems. • Understands the common ways students solve Equal Sharing problems (Non-Anticipatory Sharing, Additive Coordination-Sharing one item at a time, Additive Coordination-Sharing Groups of Items, Ratio, and Multiplicative Coordination) and the common ways students solve Multiple Groups problems (Represents each Group with Direct Modeling, Represents each Group with Repeated Addition, Grouping and Combining Strategies, and Multiplicative Strategies). Analyzes student work and plans instruction to encourage development of strategies based on relational thinking. • Poses questions to help students make connections among different strategies and develop understanding of relational thinking. | | <ul style="list-style-type: none"> • Expects students to memorize procedures for solving problems with fractions. May teach a mnemonic to help students remember procedures. • Expects students to solve the fraction problems with one known procedure. • Accepts one solution strategy when multiple approaches lead to correct solutions. Focuses on correct answers rather than relational thinking. | |

| Component 3: The teacher structures the learning so students <i>construct viable arguments and critique the reasoning of others.</i> | | | |
|--|---|--|--|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> • Expects students to explain their reasoning, justify their solutions, and provide logical arguments. • Facilitates student collaboration in order for students to listen to and evaluate the solution strategies of others. • Provides opportunities to compare the effectiveness of plausible arguments and distinguish correct and flawed reasoning. This includes presenting incorrect answers and non-examples for students to evaluate when they do not occur in class discussions. | <ul style="list-style-type: none"> • Asks leading questions to help students explain their reasoning. • Highlights strategies for students to share with others during whole class discussion. • Discusses arguments and asks students to distinguish correct and flawed reasoning when it occurs in class. Occasionally shares incorrect reasoning when it doesn't occur in class and explains why it is wrong. | <ul style="list-style-type: none"> • Asks students to show their work, but not justify solutions. • Explains answers and solution strategies for students during whole class discussion. • Explains correct and flawed reasoning when it occurs in class. | <ul style="list-style-type: none"> • Accepts student answers without work. • Does not provide time for student collaboration or whole class discussion. • Focuses on answers rather than reasoning. |
| <ul style="list-style-type: none"> • Expects students to explain their reasoning and justify their solutions to a variety of contextual problems (Equal Sharing, Multiple Groups, and Partial Groups problems) and non-contextual open number sentences. • Facilitates student discussions of different solution strategies. Plans student presentations of strategies from concrete to symbolic in order to build understanding of relational thinking. • Provides opportunities to compare answers to deepen understanding and distinguish correct and flawed reasoning. For example, problems where multiple equivalent fractions or combinations of fractions are common answers ($1 + \frac{1}{2} + \frac{1}{4}$ or $1\frac{3}{4}$ or $\frac{7}{4}$) lead to a rich discussion of fraction equivalence and addition with unlike denominators. | | <ul style="list-style-type: none"> • Accepts student answers without written or verbal explanations. • Shows one way to solve problems, so there are not multiple strategies to discuss. • Collects and evaluates students' answers without a class discussion. | |

| Component 4: The teacher structures the learning so students <i>model with mathematics</i> . | | | |
|--|--|---|--|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> Provides opportunities for students to create mathematical models to make sense of everyday life, society, and workplace problems. Expects students to create mathematical models to show relationships among quantities. Challenges students to analyze and interpret their mathematical models in order to draw conclusions, possibly improving the model if necessary. | <ul style="list-style-type: none"> Leads discussion to help students create a predetermined model in order to understand everyday life, society, and workplace problems. Asks leading questions to help students use mathematical models to show relationships among quantities. Asks leading questions to help students analyze and interpret a mathematical model in order to draw conclusions. | <ul style="list-style-type: none"> Shows a model to help students solve everyday life, society, and workplace problems. Uses models to show and explain relationships among quantities. Explains how to use a model to draw conclusions. | <ul style="list-style-type: none"> Demonstrates how to solve problems without modeling. Does not use models to help students make sense of problems. Does not use models to draw conclusions. |
| <ul style="list-style-type: none"> Poses contextual problems to support understanding of a variety of visual fraction models including region models, number lines, and set models. Students decide how to represent and solve problems using drawings. Encourages students, when developmentally ready, to notate their solution strategies with equations. These equations help students make their relational thinking explicit. Challenges students to explain connections among drawings, informal symbolic representations, and equations used to solve the same problem. | | <ul style="list-style-type: none"> Shows students how to represent fractions with premade manipulatives or pictures of partly shaded regions. Common premade manipulatives include fraction circles and fraction bars, both region models. Encourages students to show each step of traditional paper and pencil fraction procedures. Does not connect premade manipulatives and traditional procedures. | |

| Component 5: The teacher structures the learning so students <i>use appropriate tools strategically</i> . | | | |
|---|---|--|---|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> Provides and encourages use of a variety of physical and digital tools when solving problems. Students make decisions as to which tools to use for given situations. Provides time for students to explore and learn how to apply physical and digital tools to mathematical situations. | <ul style="list-style-type: none"> Guides students to select from a given set of tools when solving problems. Models how to use a few physical and digital tools when solving problems. | <ul style="list-style-type: none"> Selects one physical or digital tool for a given problem situation. Models one way to use one physical or digit tool for solving problems. | <ul style="list-style-type: none"> Does not allow students to use tools other than paper and pencil when solving problems. Does not model the use of tools other than paper and pencil. |
| <ul style="list-style-type: none"> Provides students with paper, pencils, and scissors for drawing, folding, and cutting when solving fraction problems. Provides time for students to decide how to represent fractions using available tools. | | <ul style="list-style-type: none"> Provides students with pre-made fraction manipulatives when solving fraction problems. Shows students how to use pre-made fraction manipulatives. | |

| Component 6: The teacher structures the learning so students <i>attend to precision</i> . | | | |
|---|---|--|---|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> • Expects students to communicate mathematical reasoning and answers verbally and in writing on a daily basis. • Engages students in discussions on the importance of conveying precise mathematical reasoning and using accurate calculations and measurements. • Expects precise use of definitions, symbols, and labels. Insists students provide numerical answers with a degree of precision appropriate for a problem situation. | <ul style="list-style-type: none"> • Expects students to communicate mathematical reasoning and answers verbally and in writing two to three times a week. • Explains the importance of and models the use of precise mathematical reasoning and accurate calculations and measurements. • Encourages precise use of definitions, symbols, labels, and numerical answers with a degree of precision appropriate for a problem situation. | <ul style="list-style-type: none"> • Provides opportunities for students to communicate verbally or in writing a few times a month. • Models precise mathematical reasoning and use of accurate calculations and measurements. • Inconsistently accepts students' answers which do not include precise definitions, symbols, labels, and numerical answers. | <ul style="list-style-type: none"> • Accepts students' answers without explanations. • Is not concerned with precision of language, calculations, or measurements. • Does not require precise use of definitions, symbols, labels, or numerical answers. |
| <ul style="list-style-type: none"> • Engages students in discussions about the accuracy of their drawings. For example, discuss how dividing a circular region into horizontal strips does not show equal-sized parts. • Engages students in discussions about the accuracy of expressions and equations. For example, discuss how linking computational steps with an equal sign such as $\frac{1}{2} \times 4 = 2 + \frac{1}{4} = 2\frac{1}{4}$ is incorrect. • Engages students in discussions about multiple equivalent forms of fractions. | | <ul style="list-style-type: none"> • Uses premade fraction manipulatives, so there is no need to discuss the accuracy of student drawings. • Focuses on the answers to fraction problems, regardless of the process for determining the solution. • Expects students to report answers in lowest terms and as a mixed number rather than an improper fraction regardless of the context of the problem. | |

| Component 7: The teacher structures the learning so students <i>look for and make use of structure</i> . | | | |
|---|--|---|---|
| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> Understands the mathematical structures connected to the content for the grade level he or she teaches and the vertical articulation across grades K-12. Engages students in discussions emphasizing connections between mathematical topics within and across mathematical domains. Structures activities to help students discern and use mathematical structures and properties in solving problems. | <ul style="list-style-type: none"> Understands the mathematical structures connected to the content for the grade level he or she teaches and a grade above and below. Engages students in discussions emphasizing connections between mathematical topics within mathematical domains. Models and explains how mathematical structures and properties connect to solving problems. | <ul style="list-style-type: none"> Understands mathematical structures connected to the content for the grade level he or she teaches. Isolates and teaches one standard at a time without connections to other mathematical ideas. Does not connect mathematical structures and properties to problems. | <ul style="list-style-type: none"> Understands the mathematical procedures for the grade level he or she teaches, but not the mathematical structures underlying the procedures. Isolates and teaches one procedure at a time without considering details of a given standard. Is not concerned with the mathematical structures and properties connected to the mathematical content he or she teaches. |
| <ul style="list-style-type: none"> Understands relational thinking and how the properties of operations apply to fraction computation. Plans activities to make students' relational thinking explicit. This may involve representing students' diagrams or informal symbolic work with equations or connecting one student's diagram to another student's equation. | | <ul style="list-style-type: none"> Understands the mathematical procedures for solving fraction problems, but not how relational thinking connects to procedures. Models how to use traditional procedures to solve problems and assigns practice problems. | |

| Component 8: The teacher structures the learning so students <i>look for and express regularity in repeated reasoning</i> . | | | |
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| a | b | c | d |
| Exemplary ← | | → Unsatisfactory | |
| The teacher: | | | |
| <ul style="list-style-type: none"> Engages students in looking for repeated reasoning in calculations and a problem's solution. Urges students to look for general methods and shortcuts in order to develop efficient methods to solve problems. Continually ask, "Will this always work?" and "Why does this work?" Requires students to evaluate the reasonableness of results throughout the problem solving process. | <ul style="list-style-type: none"> Directs students in looking for repeated reasoning in calculations and a problem's solution. Challenges students to find general methods and shortcuts, but explains to students why methods work. Requires students to evaluate the reasonableness of their end result. | <ul style="list-style-type: none"> Explains repeated reasoning in calculations and a problem's solution. Presents shortcuts and explains to students why the shortcut works. Evaluates the reasonableness of results and tells students to check unreasonable results. | <ul style="list-style-type: none"> Does not identify repeated reasoning in calculations and a problem's solution. Presents shortcuts without helping students understand why the shortcut works. Evaluates answers and does not address unreasonable results. |
| <ul style="list-style-type: none"> Structures instruction so students determine a generalizable method to find multiple equivalent fractions. Structures instruction so students determine a few generalizable strategies to compare or order fractions. Structures instruction so students determine the need for like denominators when adding and subtracting fractions. Structures instruction so students determine a generalizable method to multiply and divide fractions. | | <ul style="list-style-type: none"> Models how to multiply the numerator and denominator by the same non-zero whole number to determine equivalent fractions and assigns practice problems. Models how to compare fractions by finding a common denominator and assigns practice problems. Models how to find a common denominator in order to add and subtract fractions and assigns practice problems. Models how to use the standard algorithm to multiply and divide fractions and assigns practice problems. | |