

## Professional Development Module

**Title:** Teaching Fractions in Grades 3 - 6

**Content and Instructional Shifts:** K-5

**Targeted Audience:** Teachers in grades 3-6

**Grade Span:** 3-6

**Description:** Instructor notes; handouts; implementation assignments – based on *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

**Delivery time:** Session 8 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

## Session 8 Instructor Notes:

### Learning Goals:

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics*.
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

### Success Criteria:

- Teachers will classify addition and subtraction situations for fractions according to *Iowa Core Mathematics* Table 1 (page 92).
- Teachers will explain how to solve problems involving multiplication and division of fractions.
- Teachers will plan and implement a lesson on fraction computation.

**Time:** 3 hours

### Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Fraction Standards"
- Handout "Iowa Core Mathematics Tables 1 & 2 Problem Situations"
- Handout "Addition and Subtraction Problem Situations"
- Handout "Partial Groups Problem Situations for Division"
- Handout "Session 8 Assignment Sheet"

**Session 8 Activity 1**  
**Addition and Subtraction Word Problems for Fractions**

**Approximate Time:** 45 minutes

**Key Purpose:** To recognize there are a variety of problem situations for addition and subtraction of whole numbers and fractions.

**Materials:**

- Book *Extending Children’s Mathematics: Fractions and Decimals* by Empson and Levi
- Handout “Iowa Core Mathematics Tables 1 & 2 Problem Situations”
- Handout “Addition and Subtraction Problem Situations”

Activity Description	Key Discussion Points
<p><b>1. Addition and Subtraction Problem Situations</b>            Have participants locate “Iowa Core Mathematics Tables 1 &amp; 2 Problem Situations” (handout from session 1) and study the problem situations for addition and subtraction. Discuss the following questions as a whole class:</p> <ul style="list-style-type: none"> <li>• What is the difference between the first two rows of problems?</li> <li>• What is the difference among the columns for Add to and Take from problems?</li> <li>• How do Put Together/Take Apart problems differ from Add to and Take from problems?</li> <li>• How are Compare problems different from all the others in the table?</li> <li>• Which problem situations do you think tend to be most difficult for children?</li> </ul>	<p><b>1. Addition and Subtraction Problem Situations</b>            This is a short introduction to the problem situations. This will be familiar to teachers who have received CGI training. The purpose is for teachers to be aware of the different problem situations and know they exist. If you, as the facilitator, are a CGI trainer, note this is not nearly as extensive as CGI training. The focus is not learning all the problem types, but developing an awareness of their existence.</p> <ul style="list-style-type: none"> <li>• All of the problems in the first row involve a joining action, while the problems in second row involve a taking away action. These problems are also called Join and Separate problems.</li> <li>• The unknown quantity in the problem varies among the three columns.</li> <li>• Put Together/Take Apart problems do not have a joining or taking away action because it involves a static relationship among the quantities. These are also referred to as Part-Part-Whole problems.</li> <li>• Compare problems involve two different sets of items that are not combined in any way. The two sets are being compared.</li> <li>• Add to-Result Unknown, Put Together/Take Apart -Total Unknown, and Take from-Result Unknown problems tend to be easiest. Add to-Start Unknown, Take from-Start Unknown, and Compare problems tend to be more difficult.</li> </ul> <p>It is important for teachers to understand this classification of problem situations, so they are sure to give their students opportunities to solve all of the different types of problems. It is not necessary for students to identify the different problem situations.</p>

## 2. Addition and Subtraction Problem Situations for Fractions

Tell participants the same problem situations apply to fractions. Pass out “Addition and Subtraction Problem Situations” (handout). Note the problems are from *Extending Children’s Mathematics*. Have participants work with a partner to classify each problem according to a problem situation from Table 1 of *Iowa Core Mathematics*. Discuss the results as a whole class. Ask the following questions as you discuss the first four problems:

- A. How might you change this problem to make it a Change Unknown or Start Unknown?
- B. How might you change this problem to make it a Result Unknown or Start Unknown?
- C. How might you change this problem to make it a Bigger Unknown or Smaller Unknown?
- D. How might you change this problem to make it an Addend Unknown?

## 2. Addition and Subtraction Problem Situations for Fractions

### A. Take from – Result Unknown

- You could make this a Change Unknown problem by saying: Lupita started with  $2\frac{5}{6}$  packages of clay and has  $\frac{1}{3}$  package of left. How much clay did she use?
- You could make this a Start Unknown problem by saying: Lupita used  $\frac{1}{3}$  package of clay and has  $2\frac{1}{2}$  packages left. How much clay did she have when she started?

### B. Add to – Change Unknown (action and change over time)

- You could change this to a Result Unknown problem by saying: Lisa hiked  $2\frac{3}{4}$  miles in the morning and  $\frac{3}{4}$  mile in the afternoon. How many total miles did Lisa hike?
- You could change this to a Start Unknown problem by saying: Lisa is going on a  $3\frac{1}{2}$  mile hike. She has  $\frac{3}{4}$  mile left to hike. How far has she already hiked?

### C. Compare – Difference Unknown

- You could change this to a Bigger Unknown problem by saying: Nick is  $67\frac{3}{4}$  inches tall. Eric is  $1\frac{3}{4}$  inches taller than Nick. How tall is Eric?
- You could change this to a Smaller Unknown problem by saying: Eric is  $69\frac{1}{2}$  inches tall. He is  $1\frac{3}{4}$  inches taller than Nick. How tall is Nick?

### D. Put Together/Take Apart – Total Unknown

- You could change this to an Addend Unknown problem by saying: A fence post is  $5\frac{1}{2}$  feet tall. Four feet of the post is above ground. How much is below ground?

### E. Part 1: Put Together/Take Apart – Total Unknown (burritos are not joined), Part 2: Take from – Result Unknown

### F. Take from – Change Unknown

### G. Part 1: Put Together/Take Apart – Total Unknown (syrup is not joined), Part 2: Put Together/Take Apart – Total Unknown, Part 3: Compare to 1 cup

### H. Put together/Take Apart – Total Unknown with 4 addends

- I. Compare – Difference Unknown
- J. Compare – Difference Unknown

**Session 8 Activity 2**  
**Multiplication and Division of Fractions**

**Approximate Time:** 75 minutes

**Key Purpose:** To develop a robust understanding of multiplication and division of fractions with partial groups.

**Materials:**

- Book *Extending Children’s Mathematics: Fractions and Decimals* by Empson and Levi
- Handout “Partial Groups Problem Situations for Division”

Activity Description	Key Discussion Points
<p><b>1. Partial Groups Problems</b> Divide participants into groups of three or four teachers. Have participants turn to <i>Extending Children’s Mathematics</i>, pages 212-213. As a group select one problem from each of the following categories:</p> <ul style="list-style-type: none"> <li>• Partial Groups Problems – Multiplication</li> <li>• Partial Groups Problems – Measurement Division</li> <li>• Partial Groups Problems – Partitive Division</li> </ul> <p>Have participants discuss why each problem fits the given category and solve each problem two different ways.</p> <p>Have each group share their reasoning for at least one of the problems.</p>	<p><b>1. Partial Groups Problems</b> Partial Groups problems tend to be difficult to understand, so this activity gives participants another opportunity to differentiate among the three categories and solve problems. Be sure participants identify the number of groups, amount per group, and total for each problem.</p> <ul style="list-style-type: none"> <li>• When you know the number of groups and amount per group, and you need to find the total, it is a multiplication problem.</li> <li>• When you know the number of groups and total, and you need to find the amount per group it is a partitive division problem.</li> <li>• When you know the amount per group and the total, and you need to find the number of groups, it is a measurement division problem.</li> </ul>
<p><b>2. Division Algorithm</b> Part of the Session 7 homework assignment was to select one problem from “Partial Groups Problem Situations Division” (handout from Session 7) and use the problem to make sense of the traditional algorithm for dividing fractions. Ask select participants to share their work with the whole class.</p>	<p><b>2. Division Algorithm</b> The purpose of this activity is to help participants understand why the standard division algorithm works and acknowledge there is not a simple way to explain the division algorithm. Allow several participants to show what makes sense to them. An example of a partitive division problem with an explanation of the division algorithm follows.</p> <p>Problem: I have <math>\frac{1}{2}</math> cup of sugar. I have enough sugar to make <math>\frac{3}{4}</math> of a batch of punch. How much sugar is needed for a full batch?</p> <p>To solve this problem you need to know how much sugar is needed for one batch. You might think <math>(\frac{3}{4} \times ? = \frac{1}{2})</math> or <math>(\frac{1}{2} \div \frac{3}{4} = ?)</math>. If <math>\frac{1}{2}</math> cup is needed</p>

for  $\frac{3}{4}$  batch, I can divide  $\frac{1}{2}$  by 3 to find how much sugar is in  $\frac{1}{4}$  batch ( $\frac{1}{2} \div 3 = \frac{1}{6}$ ). If  $\frac{1}{4}$  batch is  $\frac{1}{6}$  cup, then a full batch is 4 times as much ( $\frac{1}{6} \times 4 = \frac{4}{6}$ ). You can divide by 3 and multiply by 4.  $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6}$ .

**Session 8 Activity 3**  
**Instructional Guidelines for Fraction Computation**

**Approximate Time: 50**

**Key Purpose:** To plan next instructional steps and prepare for Implementation Assignment 7.

**Materials:**

- Book *Extending Children’s Mathematics: Fractions and Decimals* by Empson and Levi
- Handout “Iowa Core Mathematics Fraction Standards”

**Activity Description**

**Key Discussion Points**

**1. Review Instructional Guidelines**

Place participants in grade-alike groups. Have participants read pages 217-222 of *Extending Children’s Mathematics* and then discuss the recommendations for their grade level.

**1. Review Instructional Guidelines**

The guidelines from *Extending Children’s Mathematics* do not directly align with *Iowa Core Mathematics* at all grade levels. *Iowa Core Mathematics* explicitly addresses addition and subtraction of fractions at grade 4, multiplication of fractions at grades 4 and 5, and division of fractions at grades 5 and 6. Equal Sharing problems are Multiple Groups partitive division problems. The reason for posing these problems in grades 3 and 4 is to develop conceptual understanding of fractions. You may want to refer back to “Equal Sharing Problems and Iowa Core Mathematics” (handout from Session 2) to revisit how Equal Sharing problems connect to *Iowa Core Mathematics* at grades 3, 4 and 5.

**2. Plan Next Instructional Steps**

Ask groups to consider the following questions and determine their next steps:

- What are the Iowa Core expectations on fraction computation for the grade you teach?
- What experiences have your students had with addition, subtraction, multiplication, or division of fractions?
- What problem(s) will you pose? Why?
- What number choices will you use in the problem? Why?

**2. Plan Next Instructional Steps**

- There are no expectations for fraction computation at grade 3 in *Iowa Core Mathematics*. However, students are capable of solving many of the problems provided in *Extending Children’s Mathematics*.
- Children need multiple experiences to develop a deep understanding of Iowa Core fraction standards.
- Teachers should consider the grade level they teach and the level of understanding demonstrated by their students to decide what problem(s) will move their students to a deeper understanding of fraction computation.

**Session 8 Activity 4  
Assignment**

**Approximate Time:** 10 minutes

**Materials:**

- Handout “Session 8 Assignment Sheet”

Activity Description	Key Discussion Points
<p>1. Reading Assignment:</p> <ul style="list-style-type: none"> <li>• <i>Extending Children’s Mathematics</i>, Chapter 9 (pp. 224-232)</li> <li>• “Iowa Core Mathematics Content and Practice Shifts Grades K-5”, the shifts under “Fraction Computation” (pp. 12-15)</li> </ul> <p>2. Implementation Assignment 7:</p> <ul style="list-style-type: none"> <li>• Pose one problem to address fraction computation. See pages 217-222 of <i>Extending Children’s Mathematics</i> for guidelines.</li> <li>• Facilitate a discussion of the problem based on student work.</li> <li>• Bring your students’ work with you to Session 9. Be prepared to share the following for the problem you posed to students:               <ul style="list-style-type: none"> <li>○ What problem and number choices did you use?</li> <li>○ What did you notice about your student’s thinking?</li> <li>○ What did you discuss as a class?</li> <li>○ What did you learn as a teacher?</li> <li>○ What is the next problem you will pose?</li> </ul> </li> </ul> <p>3. Item needed for Session 9:</p> <ul style="list-style-type: none"> <li>• Bring a laptop in order to access and review online assessment items from Partnership for Assessment of Readiness for College and Careers (PARCC) and Smarter Balanced Assessment Consortium (SBAC).</li> </ul>	<p>This assignment is similar to past assignments. It includes a reading assignment and an implementation assignment.</p>