Professional Development Module

Title: Teaching Fractions in Grades 3 - 6

Content and Instructional Shifts: K-5

Targeted Audience: Teachers in grades 3-6

Grade Span: 3-6

Description: Instructor notes; handouts; implementation assignments – based on *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

Delivery time: Session 6 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

Session 6 Instructor Notes:

Learning Goals:

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics.*
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

Success Criteria:

- Teachers will identify fundamental ideas related to fraction equivalency.
- Teachers will use efficient and flexible strategies to order fractions by reasoning about their size.
- Teachers will plan and implement a lesson to develop student understanding of fraction equivalency or order.

Time: 3 hours

Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Fraction Standards"
- Handout "Ordering Fractions"
- Handout "Session 6 Assignment Sheet"
- Instructor Resource "NAEP Problem and Results"
- Student work collected by each participant throughout the course

Session 6 Activity 1
Analyze Student Work from Implementation Assignment 5

Approximate Time: 30 minutes
Key Purpose: To reflect on teacher actions during the last implementation assignment and determine next steps in making relational thinking explicit.
Materials:

- Student work collected by each participant

Activity Description	Key Discussion Points
Analyze Lesson on Relational Thinking Place participants into small groups of three to four teachers. Teachers posed the following problem to their students for implementation assignment 5: It takes ____ yard of ribbon to make a bow. How many bows can I make with____ yards? $(\frac{1}{4}, 12)$ $(\frac{1}{3}, 18)$ $(\frac{1}{3}, 12\frac{2}{3})$ $(\frac{3}{4}, 12)$ $(\frac{2}{3}, 18)$ $(\frac{5}{6}, 35)$ Have each participant share: • the numbers he or she used for the problem, • the notation the student(s) used or the teacher posed, • connections made between notation and student work, and • a description of the class discussion. •	**Analyze Lesson on Relational Thinking** The purpose of this activity is for teachers to discuss what they did to develop students' understanding of relational thinking. As participants work in groups, note examples of evidence of students' understanding of relational thinking. Ask select teachers to share their student's work and thinking with the entire class.

Session 6 Activity 2
Fraction Equivalence

Approximate Time: 40 minutes
Key Purpose: To develop understanding of fraction equivalence.
Materials:

- Handout "Iowa Core Mathematics Fraction Standards"
- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Instructor Resource "NAEP Problem and Results"

Activity Description	Key Discussion Points
1. Iowa Core Mathematics Fraction Standards Have participants read standards 3.NF.A.3abcd and 4.NF.A.1-2 from "Iowa Core Mathematics Fraction Standards" (handout from Session 1). These standards s address fraction equivalence and ordering. Pose	**1. Iowa Core Mathematics Fraction Standards** Standards 3.NF.A.3abcd and 4.NF.A.1-2 address fraction equivalency and order. We hope participants will notice some of the following: • The standards stress visual fraction models including the number

the following questions: What is new, interesting, or unique? Is there anything you question or find troublesome? What are some of the most important ideas?	line. Highlight the reasons for having children create visual fraction models rather than use pre-made manipulatives (pages 116-119 of *Extending Children's Mathematics*). This is part of the reading assignment following Session 6. • Standard 3.NF.A.3b states students generate equivalent fractions. This is before standard 4.NF.A.1 where students explain why $\frac{a}{b} = \frac{a \times n}{b \times n}$. Note the progression of building conceptual understanding before learning the procedure and the emphasis on understanding and explaining the procedure rather than memorizing it. • Fraction comparisons are only valid when fractions refer to the same whole. Share "NAEP Problem and Results" (instructor resource) to show how this idea was assessed on a National Assessment of Education Progress (NAEP) test. • The emphasis on comparing fractions is by reasoning about their size. The specific strategies for comparing fractions include creating common numerators, creating common denominators, and using benchmark fractions such as $\frac{1}{2}$ and $\frac{1}{4}$.
2. Classroom Discussion on Fraction Equivalence The assignment for this session included the following: Bring student work from an Equal Sharing problem you have posed that resulted in students getting different equivalent answers (if you have examples). Pose the following questions and have participants share his or her classroom experiences. • Have you discussed fraction equivalency as a result of students getting different correct answers for Equal Sharing problems? • What prompted the discussion? Share the specific problem and student answers. • What numbers did you use in the problem that allowed students to get multiple equivalent answers? • What evidence of student understanding resulted from the class discussion? An alternative activity is to collect and show past student work from	**2. Classroom Discussions on Fraction Equivalence** Have several teachers share classroom experiences on fraction equivalency. We hope participants note the following: • Participants recognize discussions of equivalent fraction occurred when students solved and shared solutions to some Equal Sharing problems. • Participants recognize discussions from previous sessions on how different number choices for Equal Sharing problems provide opportunities to discuss fraction equivalence. Participants may want to refer back to "Number Choices in Equal Sharing Problems" (handout from Session 2) and revisit problem 4 from the handout. "Number Choices in Equal Sharing Problems" problem 4 states: *Children could get multiple correct answers to the brownie problem ($2\frac{1}{2}$, $2\frac{2}{4}$, or $\frac{10}{4}$). A discussion of these correct answers extends the learning to equivalent fractions. Rewrite the brownie problem using different numbers, so there will be multiple answers equivalent to $\frac{2}{3}$.*

the class where students found multiple equivalent answers for the same problem. Discuss the numbers in the problem that allowed students to get multiple equivalent answers and the questions a teacher might pose to elicit a class discussion of equivalent fractions.	
3. Key Ideas on Fraction Equivalence Have participants develop a list of fractions equivalent to $\frac{1}{3}$. Display a combined list of fractions and pose the following questions: • How many different fractions are equivalent to $\frac{1}{3}$? • How do you know all of these fractions are equivalent?	**3. Key Ideas on Fraction Equivalence** These questions address two key concepts related to fraction equivalency. • How many different fractions are equivalent to $\frac{1}{3}$? Any fraction has an infinite number of equivalent forms that equal the same amount. Most teachers understand any fraction has an infinite number of equivalent forms and all equivalent fractions represent the same amount. However, teachers often place so much emphasis on simplifying fractions students begin to think a fraction not in lowest terms is incorrect. Using the word "reduce" also causes students to think equivalent fractions are not the same amount. • How do you know all of these fractions are equivalent? There are two ways to think about fraction equivalence. These ideas may be new to many teachers. The purpose here is primarily to enhance teachers' understanding of fractions. o Scale factor: Teachers may describe the procedure for creating equivalent fractions. The procedure for creating equivalent fractions is usually thought of as scaling up or down (multiplying the numerator and denominator by the same number). The scale factor between pairs of equivalent fractions varies. For example, the scale factor between $\frac{2}{6}$ and $\frac{4}{12}$ is not the same as the scale factor between $\frac{2}{6}$ and $\frac{6}{18}$. o Constant of Proportionality: The multiplicative relationship between the numerator and denominator remains the same for all equivalent fractions. Consider $\frac{4}{12}$ and $\frac{6}{18}$. How do you know these two fractions are equivalent? The denominator is always 3 times the numerator or the numerator is always $\frac{1}{3}$ the denominator. This multiplicative relationship between the numerator and denominator is the same for all fractions

	equivalent to the unit fraction 1/3. In later grades this relationship is described as the constant of proportionality or the slope.

Session 6 Activity 3
Fraction Order

Approximate Time: 30 minutes
Key Purpose: To develop strategies to compare fractions by reasoning about their size.
Materials:

- Handout "Ordering Fractions"

Activity Description	Key Discussion Points
Comparing Fractions Have participants complete "Ordering Fractions" (handout) with a partner. The directions state participants should compare each of the following pairs of fractions mentally and **not** use common denominators, pictures, cross products, decimal fractions, or percent. 1. $\frac{2}{5}$ $\frac{5}{8}$ 2. $\frac{3}{4}$ $\frac{3}{5}$ 3. $\frac{7}{8}$ $\frac{5}{6}$ 4. $\frac{2}{5}$ $\frac{1}{3}$ 5. $\frac{9}{8}$ $\frac{6}{5}$ 6. $\frac{7}{4}$ $\frac{13}{10}$ 7. $\frac{2}{12}$ $\frac{3}{8}$ 8. $\frac{9}{12}$ $\frac{7}{8}$ Have participants share strategies for solving each pair of fractions	**Comparing Fractions** The purpose of this activity is to develop flexibility with a variety of strategies. *Iowa Core Mathematics* standards 3.NF.A.3d and 4.NF.A.2 specifically set expectations for a variety of strategies for ordering fractions. Finding a common denominator to compare fractions is not a bad strategy, but less efficient for many pairs of fractions. The following explanations describe one way to compare the fractions by reasoning about their size. Participants may share additional strategies. 1. Compare to one half. Five-eighths is greater than 1 half and 2 fifths is less than 1 half, so 5 eighths is larger. 2. Common numerators: Fourths are greater than fifths, so 3 fourths is greater than 3 fifths. 3. Compare to one. Both fractions are one part less than one. One eighth is less than 1 sixth, so 7 eighths is missing a smaller part and closer to 1. 4. Common numerators. One third is equal to 2 sixths. Fifths are greater than sixths, so 2 fifths are greater than 2 sixths. 5. Compare to one. Nine-eighths is 1 eighth greater than one and 6 fifths is 1 fifth greater than 1. One-fifth is greater than 1 eighth, so 6 fifths is greater. 6. Common numerators. Seven-fourths is equal to 1 and 3 fourths and 13 tenths is equal to 1 and 3 tenths. Fourths are greater than tenths, so 1 and 3 fourths is greater than 1 and 3 tenths. 7. Compare to 1 fourth. Three-twelfths is equal to 1 fourth, so 2

with the whole class.	twelfths is less than 1 fourth. Three-eighths is greater than 1 fourth, so it is larger. 8. Compare to 3 fourths. Nine-twelfths is equal to 3 fourths and 7 eighths is greater than 3 fourths, so 7 eighths is larger. Finding common denominators is a useful strategy for some pairs of fractions, but tends to be overused. Students who analyze the problem situation and use an appropriate and efficient strategy for comparing fractions tend to have a deeper understanding of fractions. Students who find common denominators when more efficient strategies make sense may lack understanding of fractions. Finding a common denominator is important when adding and subtracting some pairs of fractions.

Session 6 Activity 4
Problems for Fraction Equivalence and Order

Approximate Time: 40 minutes
Key Purpose: To understand how different types of problems build understanding of fraction equivalence and order.
Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

Activity Description	Key Discussion Points
Problems for Equivalence and Order 1. Have participants turn to *Extending Children's Mathematics* pages p. 139 and read the introductory paragraph. 2. Divide participants into four groups and assign each group one set of problems from pp. 139-143 of *Extending Children's Mathematics*. The first set of problems is Equal Sharing Problems and Variations. Participants have already discussed the use of Equal Sharing problems to promote understanding of fraction equivalence, so you may not need to spend time on this set. The remaining problem sets include: • Equivalencing Problems (Compare all problems and solve problem C with different number combinations.) • Equivalencing Problems – Price and Rate Contexts (Compare all problems and solve problem D with different number combinations.)	**Problems for Equivalence and Order** The purpose for solving these problems is to deepen participants understanding of fraction equivalency and to show participants the variety of problems they might pose to students to develop understanding of fraction equivalency and order. Be sure participants discuss the following ideas during the group presentations: • It is important for teachers to solve a problem before assigning the problem to students. This helps teachers fully understand the depth of knowledge needed to solve the problem and plan questions to help students if they have difficulty. • Following are two common ways to solve the Equivalencing Problems on p. 140. Consider problem B as an example. ○ Strategy 1: Using the relationship within the situation. Students determine each child receives $\frac{3}{4}$ liter of juice and then use addition ($\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}=\frac{36}{4}$ or 9

• Open Number Sentences (Solve all problems using relational thinking.) • Comparison Problems (Compare all the problems and solve problem D with different number combinations.) Have each group solve the problems in their assigned set without using standard procedures (such as cross multiplication or using techniques to solve equations for one unknown) and answer the following questions: • What will students learn about fraction equivalence and/or order by solving and discussing the problem set? • How are the problems richer than typical problems where students write equivalent fractions or determine which of two fractions is larger? • What makes the different number combinations (or equations) vary in difficulty? 3. Have each group present a summary of their learning to the entire class. Groups should answer the above questions.	liters) or use multiplication (12 x $\frac{3}{4}$ = $\frac{36}{4}$ or 9 liters) to determine the amount of juice for 12 children. ○ Strategy 2: Using the relationship between the two situations. Students determine there are three times as many children, so you need three times as much juice (3 x 3 liters = 9 liters). • Following are examples of common strategies for Comparison Problems. Consider problem B on page 142 as an example. ○ Strategy 1: Using the relationship within the situation. Students determine each child receives $\frac{2}{3}$ bag of popcorn in the first group and $\frac{4}{6}$ bag in the second group. Students then compare the two fractions and see people in each group get the same amount. ○ Strategy 2: Using the relationship between the two situations. Students determine there are twice as many people in the second group and twice as many bags of popcorn. Therefore people in both groups get the same amount.

Session 6 Activity 5
Instructional Guidelines for Fraction Equivalence and Order

Approximate Time: 30 minutes
Key Purpose: To plan next instructional steps and prepare for Implementation Assignment 6.
Materials:
- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

Activity Description	Key Discussion Points
1. Review Instructional Guidelines Place participants in grade-alike groups. Have participants read pages 144-147 of *Extending Children's Mathematics* and then discuss the recommendations for their grade level.	**1. Review Instructional Guidelines** The Instructional Guidelines provide suggestions for posing problems similar to those participants solved during class time. The guidelines address Equal Sharing problems, Equivalencing problems, Comparison problems, and Open Number Sentences. The purpose is to discuss both problem types and fractions appropriate for given grade levels.
2. Plan Next Instructional Steps Ask participants to select one Equal Sharing problem to address fraction equivalency and at least one Equivalencing problem, Open Number Sentence or Comparison problem.	**2. Plan Next Instructional Steps** • Children need multiple experiences to develop a deep understanding of fraction equivalency and order. • The purpose is for participants to pose an Equal Sharing problem

	and facilitate a discussion of fraction equivalency and to try any of the other types of problems to learn how students respond.

Session 6 Activity 6
Assignment

Approximate Time: 10 minutes
Materials:
- Handout "Session 6 Assignment Sheet"

Activity Description	Key Discussion Points
1. Reading Assignment: • *Extending Children's Mathematics*, Chapter 6 (pp. 114-147) • "Iowa Core Mathematics Content and Practice Shifts Grades K-5", the last three shifts under "The Meaning of Fractions" (pp. 11-12) 2. Implementation Assignment 6: • Pose one Equal Sharing problem to address fraction equivalency and at least one Equivalencing problem, Open Number Sentence or Comparison problem. See pages 144-147 of *Extending Children's Mathematics* for guidelines. • Facilitate a discussion of fraction equivalency based on student work. • Bring your students' work with you to Session 7. Be prepared to share the following for the Equal Sharing problem you posed to students: o What number combinations did you use? o What equivalent fractions resulted in student work? o How did your class discussion address equivalency? Be prepared to share the following for the Equivalencing problem, Open Number Sentence or Comparison problem you posed to students: o What did you notice about your students' thinking? o What did you discuss? o What did you learn?	This assignment is similar to past assignments. The difference is the focus is on fraction equivalency and order.