Professional Development Module

Title: Teaching Fractions in Grades 3 - 6

Content and Instructional Shifts: K-5

Targeted Audience: Teachers in grades 3-6

Grade Span: 3-6

Description: Instructor notes; handouts; implementation assignments – based on *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi

Delivery time: Session 2 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

Iowa Core Mathematics: Teaching Fractions in Grades 3-6
Session 2 Instructor Notes

Learning Goals:

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics.*
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

Success Criteria:

- Teachers will classify student work for Equal Sharing problems according to the five common strategies (see *Extending Children's Mathematics* p.25).
- Teachers will describe the *Iowa Core Mathematics* fraction standards teachers can develop through the use of Equal Sharing problems.
- Teachers write Equal Sharing problems based on the needs of their students.

Time: 3 hours

Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Fraction Standards"
- Handout "Iowa Core Mathematics Content and Practice Shifts Grades K-5"
- Handout "Equal Sharing Problems and Iowa Core Mathematics"
- Handout "Number Choices in Equal Sharing Problems"
- Handout "Session 2 Assignment Sheet"
- Student work collected by each participant
- Black fine point markers (2-3 per group)
- Post-it poster paper (1 per group)

Session 2 Activity 1
Analyze Student Work from Implementation Assignment 1

Approximate Time: 60 minutes
Key Purpose: To familiarize teachers with strategies and strategy names for Equal Sharing problems.
Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Student work collected by each participant
- Black fine point markers (2-3 per group)
- Post-it poster paper (1 per group)

Activity Description	Key Discussion Points
1. Revisiting the Brownie Problem and Strategy Names Have participants turn to *Extending Children's Mathematics* page 25. Select samples of work collected during Session 1 for the brownie problem to illustrate each of the strategies described on page 25. Share one strategy at a time and ask participants to classify and name the strategy.	**1. Revisiting the Brownie Problem Strategy Names** Participants solved the brownie problem using several different strategies and read about the strategies in Chapter 1. Now we want participants to identify and name the strategies they used in Session 1. • Research shows the strategies on page 25 are common to all children for Equal Sharing problems. These are also the strategies the participants use to solve the brownie problem. • It is difficult to distinguish between **Non-anticipatory Sharing** and **Additive Coordination - Sharing Groups of Items** for the brownie problem. This is because the answer is in halves and children who use Non-anticipatory Sharing usually start by making halves. Contrast the brownie problem to the example on page 25. The example on page 25 allows us to see the difference between these two strategies. • The **Ratio** strategy is not very common as not all children use this strategy. A teacher should not expect all students to use this strategy.
2. Small Group Work Place participants in grade-alike groups of three to four teachers. Have each group look at the student work from one participant and sort the work into 3 to 6 piles of similar thinking. Then have groups take each pile and determine the strategy name based on the descriptions from *Extending Children's Mathematics* page 25. Follow this process for each participant's student work.	**2. Small Group Work** • Grade 3 teachers will probably not have all the strategies from page 25. • Teachers will probably find they have several representations of the same way of thinking or category of strategy. • By analyzing and classifying student work, teachers will find there

Have each group create a poster showing each strategy used by students at their grade level.	are a limited number of ways to solve Equal Sharing problems. • Teachers tend to categorize the standard algorithm (long division) as Multiplicative Coordination. It is important to discuss this strategy does not necessarily mean a child is using Multiplicative Coordination. A teacher would have to ask further questions to clarify a child's thinking. For example, a child who used long division and got a correct answer for 4 share 10, may not get a correct answer for 10 share 4, because they divide the smaller number into the larger number.
3. Whole Class Discussion Gather around each poster as an entire class and discuss the results, starting with grade 3 work. Ask participants to identify how thinking among students at different grade levels is similar and different. Check if participants agree with classification of student work.	3. **Whole Class Discussion** • It is likely similar strategies will appear in grades 3 through 6. • Older students who have not encountered Equal Sharing problems tend to use less sophisticated strategies than those who have had more experience with Equal Sharing problems. • This allows teachers to "see" a potential progression from grades 3 to 6. • When looking at work from grades 3 to 6, teachers will again notice there are a limited number of ways to solve Equal Sharing problems.

Session 2 Activity 2
Equal Sharing Problems and Iowa Core Mathematics

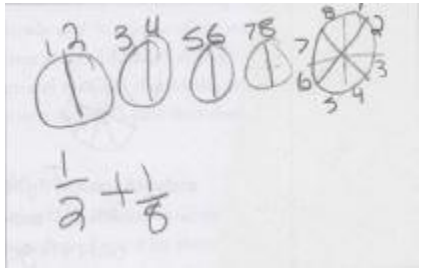
Approximate Time: 60 minutes
Key Purpose: To recognize you can help students develop the Iowa Core Mathematics fraction standards through the use of Equal Sharing problems.
Materials:
- Handout "Iowa Core Mathematics Fraction Standards"
- Handout "Equal Sharing Problems and Iowa Core Mathematics"
- Handout "Iowa Core Mathematics Content and Practice Shifts Grades K-5"

Activity Description	Key Discussion Points
1. Iowa Core Standards The homework assignment for Session 1 included the following question. Which *Iowa Core Mathematics* standards might be developed with Equal Sharing problems? Explain your thinking. Study the following three samples of student work from *Extending Children's*	**1. Iowa Core Standards** **3.NF.A.1 Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a/b* as the quantity formed by a parts of size 1/*b*.** All three students show understanding of the following ideas from standard 3.NF.A.1:

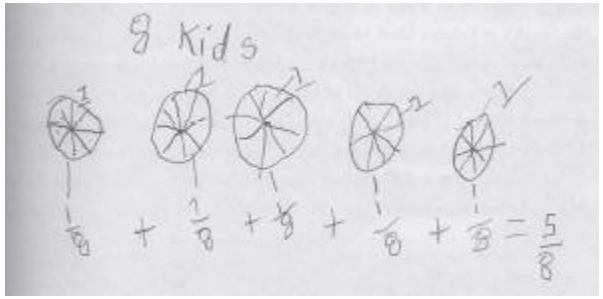
Mathematics, pages 42-43, as you discuss this question. Show each sample of student work and ask participants how the student shows evidence of understanding standard 3.NF.A.1, 4.NF.B.4a or 5.NF.B.3?

Problem: Eight people want to share five pizzas so that each person gets the same amount. How much pizza would each person get?

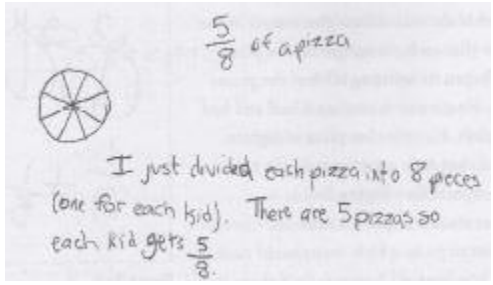
Student 1



Student 2



Student 3



Have participants complete "Equal Sharing Problems and Iowa Core

- You get fractions from dividing a whole.
- The portions need to be equal to have a fair share.
- A fraction represents a relationship between the number of parts and the size of the part. When one pizza is divided into eighths, one part is one-eighth, two parts is two-eighths, etc.
- Fractions are a quantity or amount, rather than a picture or two separate numbers (see common misconceptions, pp. 9-10 "Iowa Core Mathematics Content and Practice Shifts Grades K-5").

4.NF.B.4a Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

a. Understand a fraction *a/b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

Students who use the Additive Coordination strategy show an understanding of this standard. Student 2 used the Additive Coordination strategy and Student 3 is transitioning from Additive Coordination to Multiplicative Coordination. These students show an understanding of the following ideas:

- $1 \div 8 = \frac{1}{8}$ and $8 \times \frac{1}{8} = 1$ generalized to $1 \div n = \frac{1}{n}$ and $n \times \frac{1}{n} = 1$. Each student divides a pizza into eight parts to get eighths and shows 8 eighths is one whole pizza.
- 5 one-eighths is equal to five-eighths. Student 2 is using repeated addition, while Student 3 seems to understand $5 \times \frac{1}{8} = \frac{5}{8}$ generalized to $p \times \frac{1}{n} = \frac{p}{n}$.

5.NF.B.3 Interpret a fraction as division of the numerator by the denominator (*a/b = a ÷ b*). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get?*

Mathematics" (handout). The last row of the table is blank. Participants should complete this row by identifying additional Iowa Core fraction standards.	***Between what two whole numbers does your answer lie?*** Students who use the Multiplicative Coordination strategy show understanding of this standard. Student 3 is close to understanding the following idea: • $5 \div 8 = \frac{5}{8}$ generalized to $j \div n = \frac{j}{n}$. Participants may include the following Iowa Core fraction standards in the last row of the table. **3.NF.A.3abc Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.** **a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.** **b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.** **c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*** • Students learn about equivalent fractions by discussing and justifying multiple correct answers. For example, the brownie problem may lead a discussion of why $\frac{1}{2} = \frac{2}{4}$ and $2\frac{1}{2} = 2\frac{2}{4}$. • Students learn fractions may be equivalent to whole numbers by discussing and justifying multiple correct answers. For example, the brownie problem may lead a discussion of why $\frac{4}{4} = 1$, $\frac{8}{4} = 2\ \frac{10}{4} = 2\frac{2}{4}$. **4.NF.B.3b Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.** ***b.* Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*** • Students learn to compose and decompose fractions when they

	combine the parts for each sharer. For example, with the brownie problems students may determine $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$, $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{8}{4}$ or 2, and $\frac{1}{4}+\frac{1}{4}=\frac{2}{4}$.
2. Premade Manipulatives The homework assignment for Session 1 included the following questions: The authors recommend teachers do **not** provide children with access to premade fraction manipulatives, but rather provide blank paper, crayons, pencils, and scissors. What is the rationale for this? Do you agree with this recommendation? At some point during this session discuss the rationale for not providing students with premade manipulatives when solving fraction problems. This discussion may come up earlier in the session. If it has not been discussed, take time to address the issue.	**2. Premade Manipulatives** • Students need to understand fractions in order to use premade manipulatives in a meaningful way. Too often children focus on the number of total parts and the number of shaded parts when using premade manipulatives. This relates to the misconception that a fraction $\frac{a}{b}$ is two whole numbers rather than a single quantity. (See Iowa Core Mathematics Content and Practice Shifts Grades K-5.) We want students to focus on the relationship between the numerator and denominator ($\frac{a}{b}$ as the quantity formed by 'a' iterations of the unit fraction $\frac{1}{b}$). • When students represent problems with drawings they are thinking and coordinating ideas. They are developing understanding of relationships by focusing on the size of a part in relation to the whole. • Students' drawings help teachers understand what the students are thinking. • Some teachers may be concerned about students' imprecise representations. If students draw very imprecise representations, it is important to ask oneself whether or not the child understands their drawing is supposed to show equal-sized parts. Students learn the pieces need to be the same size by solving and discussing problems. Teachers can help children learn to make more precise drawings.

Session 2 Activity 3
Instructional Guidelines for Equal Sharing Problems

Approximate Time: 45 minutes
Key Purpose: To understand the importance of number choice when writing problems and prepare for Implementation Assignment 2.
Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Fraction Standards"

• Handout "Number Choices in Equal Sharing Problems"	
Activity Description	**Key Discussion Points**
1. Review Instructional Guidelines Place participants in grade-alike groups. Have participants read pages 32-35 *of Extending Children's Mathematics* and then discuss the recommendations for their grade level. Ask groups to consider the following questions: • How does the number of sharers relate to the answer of an Equal Sharing problem? • How does the number of sharers recommended in *Extending Children's Mathematics* guidelines compare to the denominators listed for grades 3 and 4 of *Iowa Core Mathematics* (see footnotes)?	**1. Review Instructional Guidelines** • The guidelines from *Extending Children's Mathematics* do not directly align with *Iowa Core Mathematics* at all grade levels. *Iowa Core Mathematics* does not explicitly address multiplication or division until grade 3. While children are capable of solving the problems described for grades K through 2 in *Extending Children's Mathematics*, teachers of grades 3 through 6 should not expect students to have experience with the types of problems. • The number of sharers determines the denominator for Equal Sharing problems. However, with some problems (such as the brownie problem) the answer may be written as an equivalent fraction with a smaller denominator. • Grade 3 guidelines in *Extending Children's Mathematics* recommend the number of sharers be 2, 3, 4, 6, 8 and 10 for most students. Grade 3 *Iowa Core Mathematics* limits denominators to 2, 3, 4, 6, and 8. • Grade 4 guidelines in *Extending Children's Mathematics* recommend the number of sharers be 2, 3, 4, 6, 8 and 10 as well as other numbers such ass 15, 20, and 100. Grade 4 *Iowa Core Mathematics* limits denominators to 2, 3, 4, 5, 6, 8, 10, 12, and 100.
2. Number Choices Have participants complete "Number Choices in Equal Sharing Problems" (handout) in small groups. Discuss the results as a whole class.	**2. Number Choices** Number choice is very important. It takes teachers time and experience to develop skill at selecting numbers for contextual situations to deepen students' understanding. The multiplicative relationship between the numerator and denominator determines the fractional answer. The following points correspond to the questions on the "Number Choices in Equal Sharing Problems" handout. • The number of sharers determines the size of the denominator. Four sharers results in fourths. Since two is a common factor of 10 and 4, you can also use halves. • The relationship between the numerator and denominator

	determines whether or not it will work to use repeated halving. We want teachers to recognize number choices that move students away from the non-anticipatory repeated halving strategy. For example, repeated having will not yield a correct answer with an odd number of sharers (3, 5, 7, etc.). • The number of sharers compared to the total determines if the answer will be greater than or less than one. If the number of sharers is greater than the total, the answer will be less than one. If the number of sharers is less than the total, the answer will be greater than one. • The number of sharers compared to the total determines if there will be multiple equivalent answers. When the number of sharers and the number of brownies have a common factor, multiple equivalent answers are possible. In order to get an answer of $\frac{2}{3}$, you could have 6 share 4, 9 share 6, 12 share 8, etc.
3. Plan Next Instructional Steps Ask groups to consider the following questions and determine the next steps: • How did your students do with the brownie problem? What other experiences have your students had with Equal Sharing problems? • What problem(s) will move your students to a deeper understanding of fractions? Consider the context and numbers. • What is the next problem(s) you plan to give your students? Why did you select the problem(s)?	**3. Plan Next Instructional Steps** Children need multiple experiences with Equal Sharing problems to develop a deep understanding of Iowa Core fraction standards. Two problems are not enough.
Session 2 Activity 4 Assignment	
Approximate Time: 15 minutes **Materials:** • Handout "Session 2 Assignment Sheet"	
Activity Description	**Key Discussion Points**
Pass out "Session 2 Assignment Sheet". All assignments are listed on this handout. 1. Reading Assignment: • *Extending Children's Mathematics*, Chapter 1 (pp. 32-35)	**Implementation Assignment 2:** • Participants have reviewed the guidelines provided on pages 32 to 35 of *Extending Children's Mathematics* during class. • Participants have also had time to write Equal Sharing problems

• *Extending Children's Mathematics*, Chapter 2 (pp. 36-46) • "Iowa Core Mathematics Content and Practice Shifts Grades K-5", the first three content shifts under "The Meaning of Fractions" (pp. 9-11) What connections do you see to our classroom discussions? 2. Implementation Assignment 2: • Select an Equal Sharing problem and pose it to your students without providing instruction on how to solve the problem. Read the problem with your students regardless of their grade level. Read "Equal Sharing Problems" pp. 29-31, and "Instructional Guidelines for Equal Sharing Problems and Introducing Fractions" pp. 32-35 for recommendations on what number to use. • Classify your students' work by sorting the work into 3 to 6 piles of similar thinking. Then take each pile and align the thinking to a strategy on p. 25. • Bring your students' work with you to Session 3. Be prepared to share how you classified the work and your thoughts on the following questions: o Do you think the new problem was easier or more difficult than the brownie problem? o How are the strategies your students used for the new problem similar or different from the strategies they used for the brownie problem? o Are there students who struggled with the new problem who were able to solve the brownie problem? Why do you think this happened? o What are your next steps? What problem will you give next?	during class. They may use a problem they wrote for this assignment. • Encourage participants to give their students multiple experiences with Equal Sharing problems, but only select one problem for this assignment.