

## Professional Development Module

**Title:** Teaching Fractions in Grades 3 - 6

**Content and Instructional Shifts:** K-5

**Targeted Audience:** Teachers in grades 3-6

**Grade Span:** 3-6

**Description:** Instructor notes; handouts; implementation assignments – based on *Extending Children’s Mathematics: Fractions and Decimals* by Empson and Levi

**Delivery time:** Session 1 of 10 three-hour sessions

The following materials were designed with the intent that the presenter(s) would be educators who have a deep understanding of the mathematical content being addressed at this level.

## Iowa Core Mathematics: Teaching Fractions in Grades 3-6

### Session 1 Instructor Notes

#### Learning Goals:

- Teachers will understand the content and instructional shifts for teaching fractions resulting from adoption of *Iowa Core Mathematics*.
- Teachers will understand the grade-specific expectations and cross grade-level learning progressions of the *Iowa Core Mathematics* fraction standards.
- Teachers will understand and implement research-based instructional strategies to build students' understanding of fractions and algebra.

#### Success Criteria:

- Teachers will identify key content and instructional shifts resulting from implementation of the Iowa Core fraction standards.
- Teachers will explain the potential of developing fraction understanding with Equal Sharing problems.
- Teachers will describe the difference between partitive and measurement division problems.

**Time:** 3 hours

#### Materials:

- Book *Extending Children's Mathematics: Fractions and Decimals* by Empson and Levi
- Handout "Iowa Core Mathematics Fraction Standards"
- Handout "Iowa Core Mathematics Content and Practice Shifts Grades K-5"
- Handout "Multiplication and Division Problems"
- Handout "Iowa Core Mathematics Tables 1 & 2 Problem Situations"
- Handout "Session 1 Assignment Sheet"
- Instructor Resource "Iowa Core Mathematics Fractions Standards with Highlights"
- Black fine point markers (2-3 per group)
- Unlined 8.5 by 11-inch paper (5-6 per group)
- Blocks or counters (at least 12 per person)

#### **Prior to Session 1 email participants "Iowa Core Mathematics Fraction Standards" and give the following assignment:**

Read "Iowa Core Mathematics Fraction Standards" (attached) and highlight standards or phrases indicating an approach different from typical practice. Take notes on what you find interesting, unique, challenging, and troublesome. Be prepared to discuss your thoughts.

**Session 1 Activity 1  
Introduction to Class**

**Approximate Time:** 15 minutes

**Materials:** None

Activity Description	Key Discussion Points
<ol style="list-style-type: none"> <li>Welcome participants and allow time for introductions.</li> <li>Provide information on location of restrooms, session times, lunch arrangements if full-day sessions, etc.</li> <li>Set expectations for attendance and assignments.</li> </ol>	<p>This session is packed, so keep the introduction activity short.</p>

**Session 1 Activity 2  
Introduction to Content and Practice Shifts for Fraction Instruction**

**Approximate Time:** 30 minutes

**Key Purpose:** To introduce participants to the *Iowa Core Mathematics* fraction standards and “Iowa Core Mathematics Content and Practice Shifts Grades K-5”. This is the beginning of the discussion, so do not expect to address all aspects of the documents or expect teachers to fully understand these ideas.

**Materials:**

- Handout “Iowa Core Mathematics Fraction Standards”
- Handout “Iowa Core Mathematics Content and Practice Shifts Grades K-5”

Activity Description	Key Discussion Points
<p><b>1. Content shifts</b> Show the Cluster Headings for fractions for grades 3-6 and pose the question, What content shifts result from adoption of <i>Iowa Core Mathematics</i> in the area of fractions?</p> <p><b><i>Iowa Core Mathematics</i> Cluster Headings:</b> Gr. 3. Develop an understanding of fractions as numbers. Gr. 4. Extend understanding of fraction equivalence and ordering. Gr. 4. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. Gr. 4. Understand decimal notation for fractions, and compare decimal fractions. Gr. 5. Use equivalent fractions as a strategy to add and subtract fractions. Gr. 5. Apply and extend previous understandings of multiplication and</p>	<p><b>1. Content Shifts</b></p> <ul style="list-style-type: none"> <li>• The majority of fraction computation work is in 4th and 5th grade. Before implementation of <i>Iowa Core Mathematics</i> many 6<sup>th</sup> and 7<sup>th</sup> grade teachers spent a large amount of time teaching fraction computation.</li> <li>• A deep understanding of fractions prepares students in grades 6 through 8 to study proportional reasoning and algebra.</li> <li>• First and second grade students began to develop understanding of fractions by partitioning area models into equal-sized parts. These standards are in the Geometry Domain (1.G.A.3, 2.G.A.2-3, 3.G.A.2).</li> </ul>

<p>division to multiply and divide fractions. Gr. 6. Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</p>	
<p><b>2. Content/Practice Shifts</b> Prior to session 1 participants read “Iowa Core Mathematics Fraction Standards” (handout) and took notes on what they found interesting, unique, challenging, and troublesome. You may want extra paper copies of the fraction standards available.</p> <p>Discuss: What stands out in these standards? What is new or unique? Is there anything you question or find troublesome?</p> <p>Focus on participant ideas. Pass out “Iowa Core Mathematics Content and Practice Shifts Grades K-5” (handout) and make connections to participant comments.</p>	<p><b>2. Content/Practice Shifts</b> The focus of this discussion is not all the content and practice shifts for fractions. Highlight the shifts brought out by the participants. We hope participants will notice some of the following:</p> <ul style="list-style-type: none"> <li>• emphasis on visual fraction models and the number line</li> <li>• emphasis on the unit fraction</li> <li>• emphasis on reasoning about the size of fractions</li> <li>• emphasis on composing and decomposing fractions</li> <li>• emphasis on understanding</li> <li>• etc.</li> </ul>
<p><b>3. Focus on Understanding</b> Have participants go back to “Iowa Core Mathematics Fraction Standards” (handout) and highlight the word understand or a form of understand (understands, understanding, etc.) throughout the standards and discuss the results.</p>	<p><b>3. Focus on Understanding</b> The Instructor Resource “Iowa Core Mathematics Fractions Standards with Highlights” has the word ‘understand’ and forms of ‘understand’ (understands, understanding, etc.) highlighted in yellow.</p> <ul style="list-style-type: none"> <li>• Six of the seven cluster titles contain the word ‘understand’.</li> <li>• The word ‘understand’ is used 12 times in the 18 fraction standards.</li> <li>• Share quote from the forward of <i>Extending Children’s Mathematics</i>, p. xiii: “In the long run, learning with understanding makes learning easier, is more efficient, more adaptable, and more readily retained.”</li> </ul> <p>This course will emphasize understanding fraction concepts and computation, not memorizing rules or procedures. The primary resource is the book <i>Extending Children’s Mathematics</i>. It is research based and aligns to the CGI philosophy for teaching elementary children mathematics.</p>
<p><b>Session 1 Activity 3</b> <b>Common Student Misconceptions</b></p>	
<p><b>Approximate Time:</b> 30 minutes</p>	

**Key Purpose:** To make the point that traditional fraction activities, such as shading regions leads to student misconceptions.

**Materials:**

- Handout “Iowa Core Mathematics Content and Practice Shifts Grades K-5”
- Book *Extending Children’s Mathematics: Fractions and Decimals* by Empson and Levi

Activity Description	Key Discussion Points
<p><b>1. Examples of Student Misconceptions</b> Place participants in small groups. Have participants read Holly and Ernesto’s thinking (<i>Extending Children’s Mathematics</i> page 4, paragraph 3 through page 5, paragraph 2). Have small groups discuss the following questions:</p> <ul style="list-style-type: none"> <li>• What about each child’s reasoning is incorrect?</li> <li>• What misconceptions do these children have about fractions?</li> <li>• Why might a teacher initially think each student understands fractions?</li> </ul> <p>Have small groups share the misconceptions they identified with the entire class.</p>	<p><b>1. Examples of Student Misconceptions</b> Holly:</p> <ul style="list-style-type: none"> <li>• Her drawing shows four items each split in half which is 8 halves.</li> <li>• She seems to think of 4 halves as 4 separate items or images rather than an amount.</li> <li>• She does not seem to understand she is able to combine the halves to make wholes.</li> <li>• She does not seem to understand 4 halves is the same as 2.</li> <li>• Her initial answer is correct. Her misunderstanding is revealed with further questions.</li> </ul> <p>Ernesto:</p> <ul style="list-style-type: none"> <li>• It appears he thinks 1 fourth is the process of making 4 parts.</li> <li>• He is not able to represent 1 fourth of the cookie when 1 half is shaded.</li> <li>• His initial answer is correct. He is able to represent 1 fourth when shown the whole cookie or unit.</li> </ul>
<p><b>2. Misconceptions in Shifts Document</b> Refer participants to “Iowa Core Mathematics Content and Practice Shifts Grades K-5” (handout) and discuss the common misconceptions listed in the document on pages 9 -10. These misconceptions may be the same or in addition to those already identified by the teachers.</p>	<p><b>2. Misconceptions in Shifts Document</b> Common misconceptions from “Iowa Core Mathematics K-5 Content and Practice Shifts”:</p> <ul style="list-style-type: none"> <li>• <i>Fractions are a brand new idea not connected to whole numbers.</i> Fractions are not numbers. Fractions are a type of picture.</li> <li>• <i>Fractions are always less than one.</i> Region models predominantly show fractions less than one. The language “three out of four” compounds this issue as it doesn’t make sense to have “five out of four.”</li> <li>• <i>A fraction <math>\frac{a}{b}</math> is two whole numbers rather than a single quantity.</i> Consider the fraction <math>\frac{3}{4}</math>. When the task is to shade three out of four equal-sized parts, students often think of three and four as</li> </ul>

separate quantities, rather than recognizing  $\frac{3}{4}$  is 3 one-fourths or three iterations of  $\frac{1}{4}$ . Using the language “three out of four” compounds this issue.

You may want to show a typical fraction activity sheet where students identify the fraction of a shaded region or shade to show a fractional amount of a region.

**Session 1 Activity 4  
Brownie Problem**

**Approximate Time:** 60 minutes

**Key Purpose:** Introduce Equal Sharing problems and students’ strategies for solving them.

**Materials:**

- Black fine point markers (2-3 per group)
- Unlined  $8\frac{1}{2}$  by 11-inch paper (5-6 per group)

**Activity Description**

**Key Discussion Points**

**1. Brownie Problem**

Four children want to share 10 brownies so that everyone gets exactly the same amount and there are none leftover. How much brownie can each child have?

Ask each small group to find 2 to 3 different ways students might solve the brownie problem and show each strategy on a different unlined sheet of paper using a black marker. If participants do not generate a variety of strategies consider asking them to think about children at a variety of grade levels.

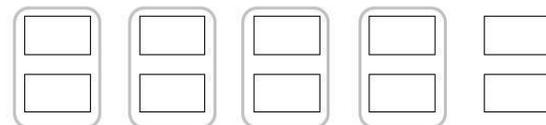
Compare and contrast the strategies as an entire class. Start the discussion by asking groups to share their least sophisticated strategy. Collect the pages for reference during Session 2.

**1. Brownie Problem**

This discussion is an introduction to strategies for solving Equal Sharing problems. Participants will learn the names of strategies and see additional examples when they read chapter 1 of *Extending Children’s Mathematics* after this session.

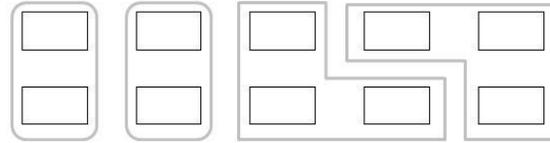
The following examples show ways participants might decide a child could solve the brownie problem. Participants’ drawings may look different, but reflect the same thinking. If participants do not show all of the strategies listed, you may want to share the additional methods or wait until Session 2 when you revisit these strategies.

- One common error children make is not sharing all the brownies. For example, a child may give each of the four children two brownies and have two left over.

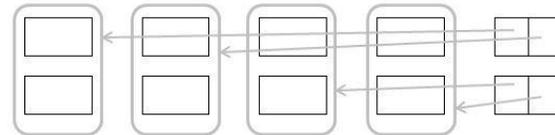


- Another common error children make is not sharing the brownies equally. For example, a child may give two children two brownies

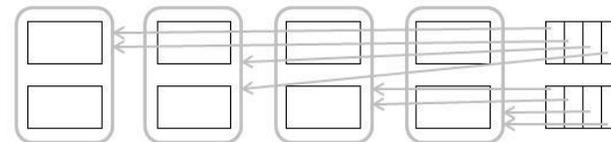
each and two children three brownies each.



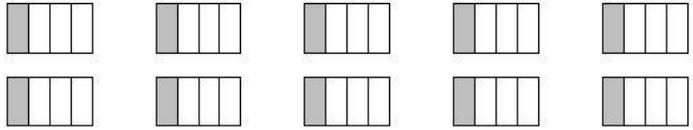
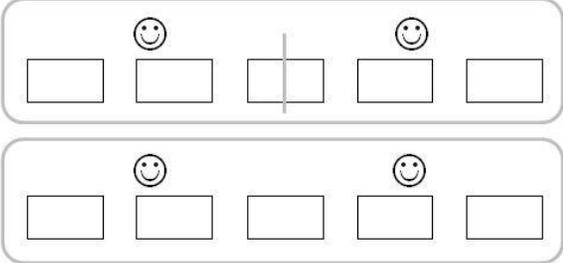
- Some children may not interpret their answers correctly due to a limited understanding of fractional names. Children may represent  $2\frac{1}{2}$  brownies, but state the answer is three, three pieces, or two and a piece. These children may not understand the partial brownie represents an amount equal to  $\frac{1}{2}$  brownie or the children may not understand the unit is one brownie.
- A child might give each person two brownies and then divide each of the remaining two brownies in half, so each person gets  $2\frac{1}{2}$  brownies. It is difficult to know whether this child understood the remaining two brownies must be divided into four equal parts or whether the child just tried halving and it worked.



- A child might give each person two brownies and then split each of the remaining two brownies in fourths, so each person gets  $2\frac{2}{4}$  brownies.



- A child might split each brownie into fourths and share the pieces. Then the child would give each person 1 fourth of each brownie for a total of 10 fourths. This is similar to the previous thinking.

	 <ul style="list-style-type: none"> <li>A child might recognize 4 sharing 10 is the same as 2 sharing 5. When two people share five they each get <math>2\frac{1}{2}</math> brownies.</li> </ul>  <ul style="list-style-type: none"> <li>A child does not need to draw a diagram because he or she understands 10 brownies shared by 4 people is <math>\frac{10}{4}</math> or <math>2\frac{1}{2}</math>. If asked to explain his or her thinking the child might draw a diagram or explain how splitting each brownie into fourths allows each person to have 1 fourth of each brownie for a total of 10 fourths.</li> <li>A child might recognize the brownie problem is a division situation and use the division algorithm to determine <math>10 \div 4 = 2.5</math> or <math>2\frac{1}{2}</math>. Whether or not this child understands fractions is inconclusive. If the child solves this problem correctly, but divides <math>8 \div 5</math> for a situation where 8 share 5, they probably do not understand.</li> </ul>
<p><b>2. Starting Fraction Instruction with the Brownie Problem</b>  The authors of <i>Extending Children's Thinking</i>, recommend starting fraction instruction with problems like the brownie problem.</p> <p>Pose the following question:  What makes the brownie problem a good problem for the first day of work with fractions?</p>	<p><b>2. Starting Fraction Instruction with the Brownie Problem</b>  What makes the brownie problem a good problem for the first day of work with fractions?</p> <ul style="list-style-type: none"> <li>Most students understand sharing and getting a fair share.</li> <li>Brownies are easy to draw and split up.</li> <li>The problem does not contain a fraction.</li> <li>The problem lends itself to halving or repeated halving to make fourths. This is easier than thirds or fifths.</li> <li>The answer is greater than 1. This helps prevent children from</li> </ul>

- forming the misconception that fractions are less than one.
- It is an extension of work with whole numbers.
- Children should have prior experience with multiplication and division story problems with whole numbers such as 4 people share 12 brownies.

**Session 1 Activity 5**  
**Multiplication and Division Problem Types**

**Approximate Time:** 30 minutes

**Key Purpose:** Recognize the differences among multiplication, partitive division, and measurement division.

**Materials:**

- Handout “Multiplication and Division Problems”
- Handout “Iowa Core Mathematics Tables 1 & 2 Problem Situations”
- Blocks or counters (at least 12 per person)

**Activity Description**

- 1. “Multiplication and Division Problems” Questions 1 & 2**  
Have the participants complete “Multiplication and Division Problems” (handout) problems 1 and 2 in small groups. Be sure participants have blocks or counters to directly model the word problems. Discuss the differences among the problems. Ask how the actions were different with each of the three problems and what equations children might use to represent each problem.
- Grandma gives 4 cookies to each of her 3 grandchildren. How many cookies are there?
  - Grandma shares 12 cookies among her 3 grandchildren. How many cookies does each grandchild get?
  - Grandma has 12 cookies and gives 3 to each grandchild. How many grandchildren are there?

**Key Discussion Points**

- 1. “Multiplication and Division Problems” Questions 1 & 2**
- When modeling the first problem, you make 3 groups of 4. The equation is  $3 \times 4 = ?$
  - It is important to emphasize the convention in textbooks in the United States is that the first factor represents the number of groups and the second factor represents the size of the groups in a multiplication grouping problem.
  - When modeling the second problem, you share the cookies one at a time forming three groups (one for the first grandchild, one for the second grandchild, one for the third grandchild, then a second cookie for the first grandchild, etc.). This is a sharing situation as you share the cookies one at a time to each child until all the cookies are distributed equally. Children might use a division or multiplication equation such as  $12 \div 3 = ?$  or  $3 \times ? = 12$  to represent the situation.
  - When modeling the third problem, you form groups of three (three for the first grandchild, three for the second, etc. until all the cookies are gone). This is a repeated subtraction situation as you repeatedly subtract 3, or repeatedly create groups of 3, until all the cookies are gone or you cannot make another group of 3.

	<p>Children might use a division or multiplication equation such as <math>12 \div 3 = ?</math> or <math>? \times 3 = 12</math> to represent the situation.</p> <ul style="list-style-type: none"> <li>Note three tells you the number of groups in problems 1 and 2, but three tells you the size of each group in problem 3.</li> </ul>																								
<p><b>2. “Multiplication and Division Problems” Questions 3 &amp; 4</b>          Have the participants complete problems 3 and 4 in small groups. Pass out “Iowa Core Mathematics Tables 1 &amp; 2 Problem Situations” (handout). Discuss the results and identify the common names for the two types of division (partitive division and measurement division). Have participants label column 2 on their handout partitive division and column 3 measurement division. Ask participants to classify the brownie problem.</p>	<p><b>2. “Multiplication and Division Problems” Questions 3 &amp; 4</b></p> <ul style="list-style-type: none"> <li><i>Iowa Core Mathematics</i> includes the two tables, one classifying common addition/subtraction problem situations and another table classifying common multiplication/division problem situations. We are only going to discuss some of the multiplication/division problem situations at this time. We will discuss other problem situations later.</li> <li>While the problems are similar, the unknown is different for each problem.</li> </ul> <table border="1" data-bbox="1066 649 1948 1130"> <thead> <tr> <th>Word Problem</th> <th>Number of Groups</th> <th>Size of Group</th> <th>Total</th> <th>Possible Equation(s)</th> <th>Problem Type</th> </tr> </thead> <tbody> <tr> <td>Grandma gives 4 cookies to each of her 3 grandchildren. How many cookies are there?</td> <td>3</td> <td>4</td> <td>Unknown</td> <td><math>3 \times 4 = ?</math></td> <td>Multiplication</td> </tr> <tr> <td>Grandma shares 12 cookies among her 3 grandchildren. How many cookies does each grandchild get?</td> <td>3</td> <td>Unknown</td> <td>12</td> <td><math>3 \times ? = 12</math> <math>12 \div 3 = ?</math></td> <td>Partitive Division</td> </tr> <tr> <td>Grandma has 12 cookies and gives 3 to each grandchild. How many grandchildren are there?</td> <td>Unknown</td> <td>3</td> <td>12</td> <td><math>? \times 3 = 12</math> <math>12 \div 3 = ?</math></td> <td>Measurement Division</td> </tr> </tbody> </table> <ul style="list-style-type: none"> <li>A division problem tells you the total and either the number of groups or the size of each group. If you know the number of groups, you can solve the problem by sharing. It is called a partitive division problem. If you know the size of each group, you can solve the problem with repeated subtraction. It is called a measurement division problem.</li> <li>The brownie problem tells you the total number of brownies (10) and the number of groups (4), so it is a partitive division problem.</li> </ul>	Word Problem	Number of Groups	Size of Group	Total	Possible Equation(s)	Problem Type	Grandma gives 4 cookies to each of her 3 grandchildren. How many cookies are there?	3	4	Unknown	$3 \times 4 = ?$	Multiplication	Grandma shares 12 cookies among her 3 grandchildren. How many cookies does each grandchild get?	3	Unknown	12	$3 \times ? = 12$ $12 \div 3 = ?$	Partitive Division	Grandma has 12 cookies and gives 3 to each grandchild. How many grandchildren are there?	Unknown	3	12	$? \times 3 = 12$ $12 \div 3 = ?$	Measurement Division
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	<p>It is a special type of partitive division called an Equal Sharing problem. An Equal Sharing problem has a whole number of groups, but the size of each group is a fraction (not equal to a whole number).</p> <ul style="list-style-type: none"> <li>Note to Instructor: At this point we are only working with Equal Groups situations from Table 2 of <i>Iowa Core Mathematics</i>.</li> </ul>
<b>Session 1 Activity 6 Assignment</b>	
<p><b>Approximate Time:</b> 15 minutes  <b>Materials:</b></p> <ul style="list-style-type: none"> <li>Handout “ Session 1 Assignment Sheet”</li> </ul>	
<b>Activity Description</b>	<b>Key Discussion Points</b>
<p>Pass out “Session 1 Assignment Sheet”. All assignments are listed on this handout.</p> <p><b>1. Reading Assignment</b>  Read <i>Extending Children’s Mathematics</i>:</p> <ul style="list-style-type: none"> <li>Foreword (pp. xi-xiv)</li> <li>Introduction (pp. xvii –xxvi)</li> <li>Chapter 1 (pp. 2-31)</li> </ul> <p><b>2. Discussion Questions:</b>  Write out answers to the following questions, so you are prepared to discuss your thoughts during Session 2:</p> <ul style="list-style-type: none"> <li>Which <i>Iowa Core Mathematics</i> standards might be developed with Equal Sharing problems? Explain your thinking.</li> <li>The authors of <i>Extending Children’s Mathematics</i> recommend teachers do <b>not</b> provide children with access to premade fraction manipulatives, but rather provide blank paper, crayons, pencils, and scissors. What is the rationale for this? Do you agree with this recommendation?</li> </ul> <p><b>3. Implementation Assignment 1</b></p> <ul style="list-style-type: none"> <li>Pose the brownie problem to your students without providing instruction on how to solve it. (Read “How to Begin” p. 11). Read the problem with your students regardless of their grade level.</li> <li>If a student solves this problem by using the standard division</li> </ul>	<p><b>Discuss Implementation Assignment 1:</b></p> <ul style="list-style-type: none"> <li>Highlight the following three general recommendations for teaching fractions from <i>Extending Children’s Mathematics</i> p. xxi: <ul style="list-style-type: none"> <li>Start with word problems and gradually incorporate equations.</li> <li>Teach without explicit instruction.</li> <li>Start with Equal Sharing problems.</li> </ul> </li> <li>If time allows, read “How to Begin”, <i>Extending Children’s Mathematics</i> page 11. If time is short, tell teachers to read this section carefully.</li> <li>Discuss the importance of posing the problem without giving instructions on how to solve it.</li> <li>Let the teachers know if students solve the brownie problem by using the standard division algorithm, they should ask them to show a second strategy.</li> </ul> <p>Note to instructors: The participants will classify their student work in small groups during Session 2.</p>

algorithm, ask him or her to solve the problem using a second strategy.

- Collect your students' work and bring it with you to Session 2.

**4. Optional Challenge Problem**

- Complete "Multiplication and Division Problem Types" challenge problem on page 2.